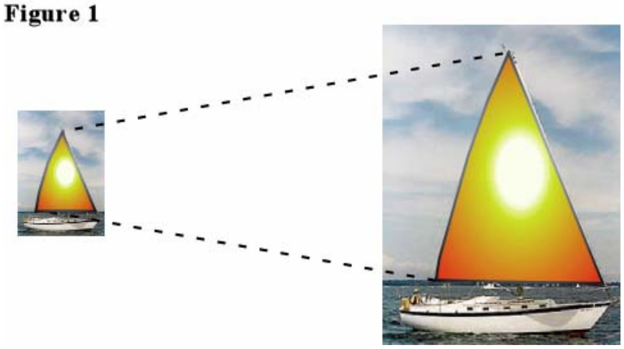
Similar Triangles

Imagine you are having a picnic at the lake. You look out across the lake and see a sailboat off in the distance … the sail is a tiny triangle to the naked eye.

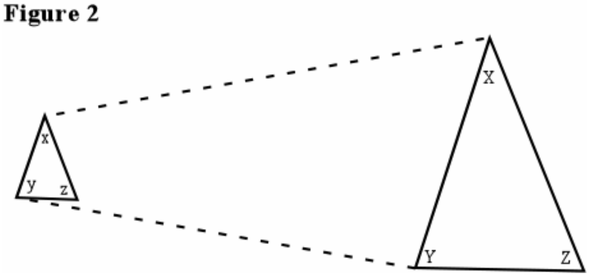
Figure 1



For a closer look, you pick up the binoculars and focus in on the sail. The sail appears to be the same shape as seen with the naked eye, only larger, as shown in Figure 1. The magnified sail is about twice as large as the small one.

Now look at a diagram of the triangular-shaped sails as shown in Figure 2. Notice that even though the sides are different lengths, the angles are the same.

Figure 2



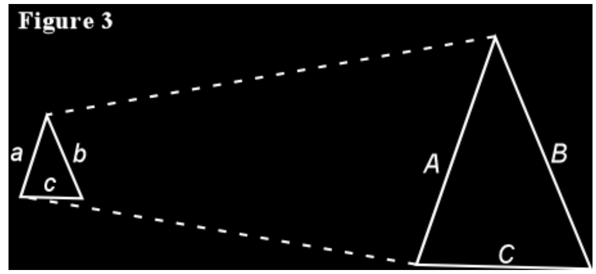
Angles x, y, and z in the little triangle are in the same relative positions as x, y, and z in the large triangle. That is to say that x, y, and z in the small triangle **correspond** to angles x, y, and z in the large triangle. Not only do these angles correspond, their measures are exactly the same in both triangles. Because the angles are the same in the two triangles, we say that these two triangles are **similar**.

This concept of **similarity** has significant mathematical importance.

- Triangles are similar if corresponding angles are equal.

Now take a look at the sides of the two triangles shown in Figure 3.

Figure 3



The sides of the little triangle are visibly shorter than the sides of the larger triangle. There is still a correspondence between the sides, however. Side *a* is in the same relative position in the little triangle as side *A* in the large triangle, and side *b* in the small triangle corresponds to *B*, and *c* corresponds to *C*.

When triangles are similar, we say that the lengths of corresponding sides are **proportional**. That is…

$$\frac{a}{A} = \frac{b}{B} \quad \text{and} \quad \frac{b}{B} = \frac{c}{C} \quad \text{and} \quad \frac{c}{C} = \frac{a}{A}$$

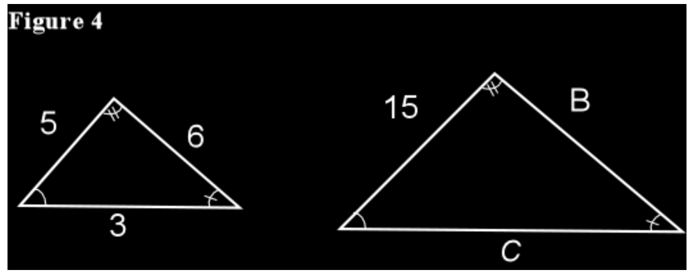
- If two triangles are similar, their corresponding side lengths are proportional.

We can use the similarity relationship to solve for an unknown side of a triangle, given the known dimensions of corresponding sides in a similar triangle.

Example 1: The two triangles in figure 4 are similar. The pairs of corresponding angles are indicated with matching hatch marks.

a) Use the similarity relationship to determine the length of side *B*.

b) Use the similarity relationship to determine the length of side *C*.



Solution:

a) Since the triangles are similar, their corresponding side lengths are proportional, so $\frac{5}{15} = \frac{6}{B}$

We solve by cross-multiplication:

$$5B = 6 \cdot 15$$
$$5B = 90$$
$$B = 18$$

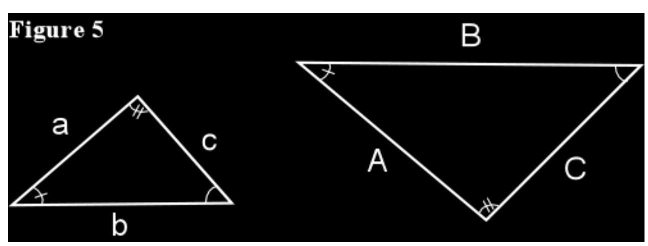
b) Likewise, using a proportion to solve for the unknown length *C*, we have

$$\frac{5}{15} = \frac{3}{C}$$
$$5C = 3 \cdot 15$$
$$5C = 45$$
$$C = 9$$

So … the sides of the larger triangle are 15, 18, and 9.

The triangles shown in Example 1 have the same orientation. That is, corresponding angles are in the same relative position. However, similar triangles may differ in their orientation. This does not change the fact that they are similar.

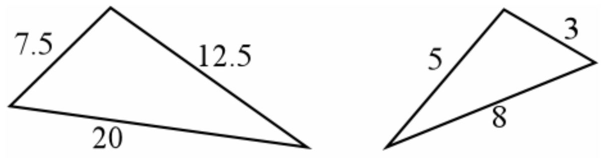
For example, the two triangles in Figure 5 have different orientations, but equal corresponding angles (as denoted by the matching hatch marks) and are, therefore, similar.



We have seen that two triangles are similar if their corresponding angles are equal. It is also true that two triangles are similar if the ratios of their corresponding sides are equal.

- If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Example 2: Use side length proportionality to determine if the triangles are similar.



Solution:

Since corresponding angles are not identified, we must determine which sides correspond. We will compare the longest side of the large triangle to the longest side of the small triangle, shortest side of the large to shortest side of the small, etc..

So… $\frac{20}{8} = 2.5$, $\frac{7.5}{3} = 2.5$, and $\frac{12.5}{5} = 2.5$

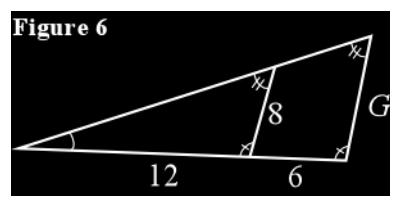
Because the ratios of corresponding sides are all equal, the side lengths are proportional, and the triangles are similar. Each side of the large triangle is exactly 2.5 times as long as the corresponding side of the small triangle.

Summary:

Similar Triangles

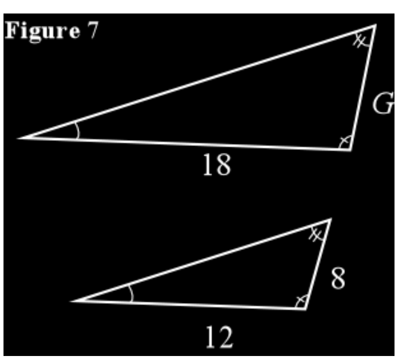
- Triangles are similar if corresponding angles are equal.
- If two triangles are similar, their corresponding side lengths are proportional.
- If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Example 3: Identify two similar triangles in Figure 6, and write a proportion to find the variable *G*.



Solution:

The two triangles overlap, sharing the marked angles as shown in Figure 7, so the triangles are similar. Note, also, that the larger triangle has a long side of 12 + 6 = 18.



The ratios of corresponding sides are equal, so …

$$\frac{G}{8} = \frac{18}{12}$$

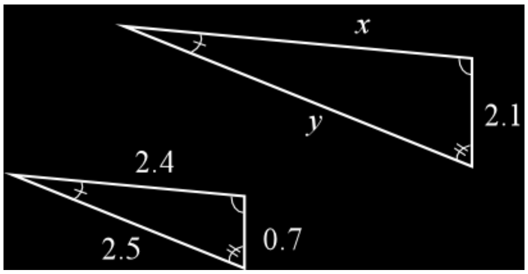
$$12G = 8 \bullet 18$$
$$12G = 144$$
$$G = 12$$

Similar Triangles

Homework:

Use the similarity relationship to find the lengths of the unknown sides of the triangles.

1.



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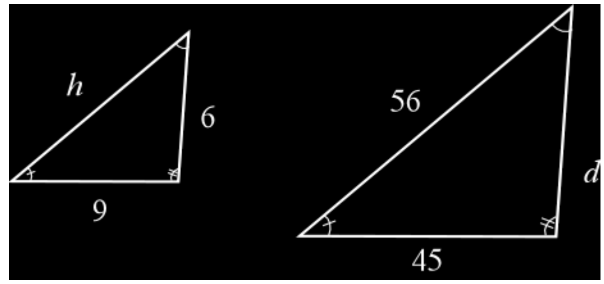

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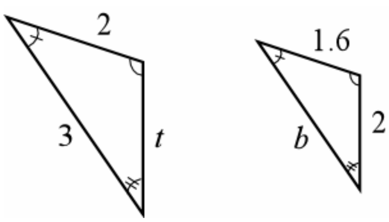

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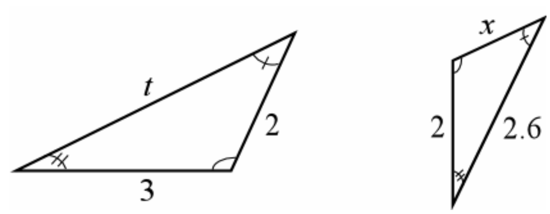

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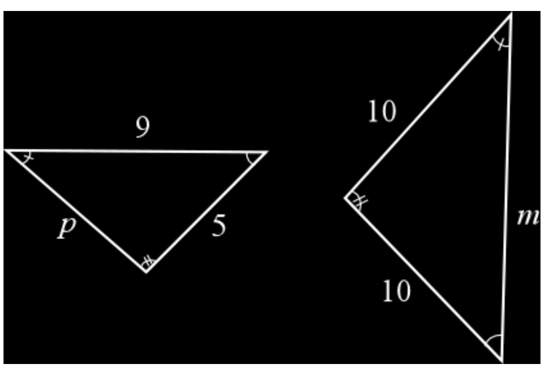

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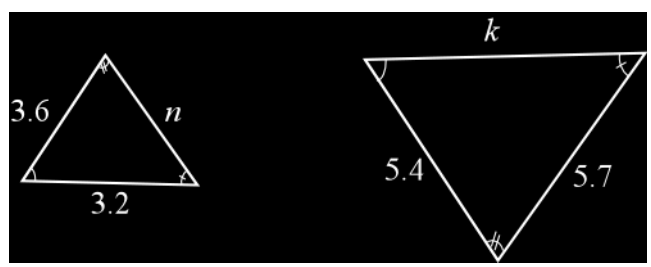
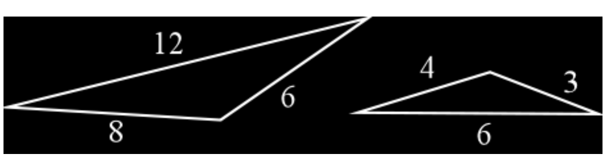


Use side length proportionality to determine if the triangles are similar.

7.



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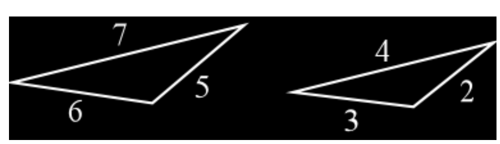
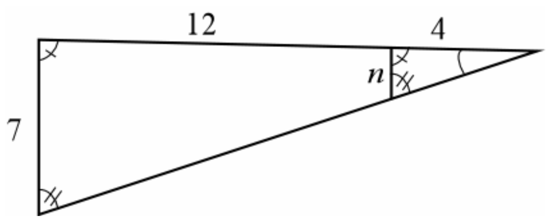


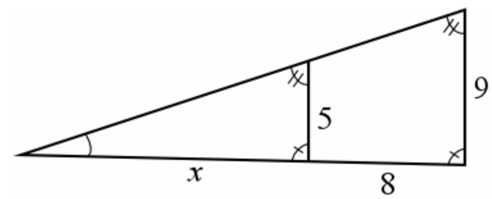
Similar Triangles

Find the unknown dimensions.

9.



10.



11. A tree casts a shadow along the ground that is 36 feet long. At the same time of day a 6-foot-tall man casts a 4.5-foot shadow. Sketch similar triangles to represent the tree and its shadow and the man and his shadow. Use the similar triangles to determine the height of the tree.

12. A 6-foot tall woman casts a 2.5-foot shadow at the same time of day that a building casts a 100-foot shadow. How tall is the building?

Answers:

1. $x = 7.2$, $y = 7.5$

2. $h = 11.2$, $d = 30$

3. $b = 2.4$, $t = 2.5$

4. $x = 4/3$, $t = 3.9$

5. $p = 5$, $m = 18$

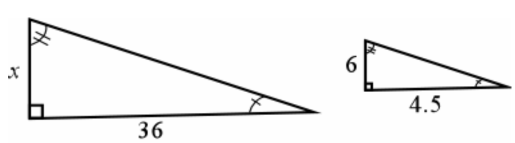
6. $n = 3.8$, $k = 4.8$

7. 12/6 = 2, 8/4 = 2, 6/3 = 2 The ratios are equal, therefore the triangles are similar.

8. 7/4 = 1.75, 6/3 = 2, 5/2 = 2.5 The ratios are not equal, therefore the triangles are not similar.

9. $n = 7/4$ or 1.75 10. $x = 10$

11.



$$\frac{x}{6} = \frac{36}{4.5}$$
$$4.5x = 6 \cdot 36$$
$$x = 48$$

12. The building is 240 feet tall.